神戸市看護大学紀要 Vol. 2, pp. 1-14, 1998

Digital Observers and Servomechanisms for Linear Dynamic Systems with Periodic Disturbances

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Abstract

A digital observer is proposed for a linear dynamic system with a pcriodic disturbance. The observer estimates the system-state with rejecting the effect of the periodic disturbance. A design method of a digital servomechanism is also presented in which the output tracking to a reference signal and the disturbance rejection can individually be treated. In this case the reference signal is not necessarily periodic. The relation between the signal generated by the proposed compensator and the disturbance is further investigated. Simulation results show the effectiveness of the design method.

Key words: Control Theory, Digital Control, Observer, Periodic Disturbance, Servomechanism

1. Introduction

When we apply a conventional observer to a linear dynamic system with a periodic disturbance, the estimated system-state is inevitably affected by the distubance. The estimated value always contains an estimation error. Since this case can be considered a kind of estimation problem for a system with an unknown input, the observer proposed for this problem[1) 2)] is available. However, the systems to which we can apply the design method are extremely restricted. Moreover, transforming a continuous-time system into a discrete-time form by a suitable sampling process, the periodic disturbance vector in the discrete-time system generally spans the space of the same dimension as that of state space. Therefore, it is impossible to apply the theory of the unknown-input observer to estimate the system-states. We propose a digital observer removing the effect of the periodic disturbance as much as possible by turning the periodicity to account. The theoretical evolution is based on the idea of repetitive control.[3)~12)]

We further investigate a digital servomechanism disturbed by a periodic disturbance applying the theoretical result obtained for the observer. It is well known as the internal model principle that output tracking with no steady-state error can be attained by adding the internal model of the exogenous signal generator in a servomechanism.[13)] However, it is sometimes difficult to realize desirable transient performance if the reference signal generator is inherently different from the disturbance generator. We cannot easily add the internal model of the periodic disturbance generator in the servomechanism to reduce the effect. If the internal model of this disturbance generator is added with that of the reference signal generator, the transient error can periodically appear for a long time whenever the reference signal nonperiodically varies. We consider a design method in which we can separately treat the output tracking to a reference signal and the disturbance rejection. In addition, the relation between the signal generated by the proposed compensator and the disturbance is discussed.

2. Discrete-time system expression

Consider the following continuous-time system:

$$\dot{x}[t]=A_c x[t]+B_c u[t]+D_c q[t] \tag{1.a}$$

$$y[t]=C_c x[t] \tag{1.b}$$

where $x[t]\in R^n$, $u[t]\in R^m$, $y[t]\in R^m$ and $q[t]\in R^p$ $(p\leq m<n)$. $q[t]$ is a periodic disturbance with period l, i.e.

$$q[t+l]=q[t]\,,\ t\geq 0 \tag{2}$$

We assume that the pair (A_c, D_c) is controllable and the period l is known.

The following discrete-time expression is derived by a sampling process of a sampling period T, where the input is assumed constant between sampling instants.

$$x(i+1)=Ax(i)+Bu(i)+d(i) \tag{3.a}$$

$$y(i)=Cx(i) \tag{3.b}$$

where

$$A=\exp(A_c T),\ B=[\exp(A_c T)-I]\,A_c^{-1}B_c,\ C=C_c \tag{3.c}$$

and

$$d(i)=\int_0^T \exp(A_c(T-\tau))D_c q[iT+\tau]d\tau \tag{3.d}$$

The discrete-time system (3) is assumed controllable and observable. Since $x(i)=x[iT]$ and $y(i)=y[iT];i=0,1,2,\cdots$, (3) represents the exact dynamic behavior of (1) at sampling instants. However, since $q[t]$ cannot be measured, it is impossible to compute (3.d). In addition, the disturbance $q[t]$ can continuously vary between sampling instants unlike the input $u[t]$. Therefore, although the dimension of the disturbance vector $q[t]$ is smaller than n, $d(i);i=0,1,2,\cdots$ generally span an n-dimensional vector space. This fact makes the perfect disturbance-rejection difficult in a digital control system.

We choose the sampling period such that (a period of the periodic disturbance)/(a sampling period) is an integer L. Therefore, the relation $d(i+L)=d(i);i=0,1,2,\cdots$ holds. We introduce two types of expression of $d(i)$ to treat the disturbance term $d(i)$ as exactly as possible in the design procedures proposed in this paper. The first expression is based on an approximation method. Let $q(i)$ be $q[iT]$. We can consider $q(i)$ to be equivalently the signal generated by the following discrete-time system:

$$\mu(i+1)=P\mu(i) \tag{4.a}$$

$$q(i)=[I_p\ 0\cdots 0]\mu(i) \tag{4.b}$$

where $\mu(i)\in R^{pL}$ and

$$P=\begin{bmatrix} 0 & I_p & 0 & \cdots & 0 \\ 0 & 0 & I_p & & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I_p \\ I_p & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{pL\times pL} \tag{4.c}$$

$I_p \in R^{p\times p}$ denotes a unit matrix. We approximate $q[iT+t]$; $0<\tau<T$ with an extrapolation function of $q(i-j)$; $j=0,1,\cdots,k$. The function is given by

$$q[iT+\tau] \cong \sum_{j=0}^{k} \omega_j[iT+\tau]\, q(i-j) \tag{5}$$

where $\omega_j[t]$ is a k-th order polynomial equation of t such that

$$\omega_j[t]\Big|_{t=(i-\alpha)T} = \begin{cases} 0\,; \alpha \neq j, \alpha = 0,1,\cdots,k \\ 1\,; \alpha = j \end{cases} \tag{6}$$

These $\omega_j[t]$;$j=0,1,\cdots,k$ can,for example,be described by applying the Lagrange form.[14] Substituting (5) into (3.d), the approximated value of (3.d) is derived as follows:

$$d(i) \cong \sum_{j=0}^{k} \Lambda_j q(i-j) \tag{7}$$

where $\Lambda_j \in R^{p\times p}$; $j=0,1,\cdots,k$, which are the functions of A_c,D_c and T.

The second description is based on the periodicity of $d(i)$ itself. Since $d(i)$ is also periodic,$d(i)$ can be considered a signal generated by the following system:

$$\eta(i+1) = \bar{P}\eta(i) \tag{8.a}$$

$$d(i) = D\eta(i) \tag{8.b}$$

where $\eta(i) \in R^L$, $D \in R^{n\times L}$ and

$$\bar{P}=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{L\times L} \tag{8.c}$$

We cannot generally know the values of $\mu(0)$ in (4) and $\eta(0)$ and D in (8). The important point that should be remarked here is that $d(i)$ can be considered the signal obeying (4) and (7) or (8).

3. State observer rejecting a periodic disturbance

We investigate a digital observer rejecting a periodic disturbance. In this section we assume that $d(i)$ is given by (4) and (7). Therefore, the discrete-time system is approximately described by

$$x(i+1)=Ax(i)+Bu(i)+\sum_{j=0}^{k}\Lambda_j q(i-j) \tag{9}$$

We introduce an observer with external inputs $\phi_j(i)\in R^p; j=0,1,\cdots,k$ given by

$$\tilde{x}(i+1)=A\tilde{x}(i)-E(C\tilde{x}(i)-y(i))+Bu(i)+\sum_{j=0}^{k}\Lambda_j\phi_j(i) \tag{10}$$

where $\tilde{x}\in R^n$ and $E\in R^{n\times m}$ The estimation error is defined by

$$\xi(i)=\tilde{x}(i)-x(i) \tag{11}$$

From (9), (10) and (11)

$$\xi(i+1)=(A-EC)\xi(i)+\sum_{j=0}^{k}\Lambda_j(\phi_j(i)-q(i-j)) \tag{12}$$

If the terms $\phi_j(i)\in R^p; j=0,1,\cdots,k$ do not exist in (12), the error system (12) is continuously driven by $q(i-j); j=0,1,\cdots,k$. We now investigate how to generate $\phi_j(i)$ to eliminate $q(i-j); j=0,1,\cdots,k$ from (12). We introduce $H\in R^{p\times m}$ whose details will be explained later. At present it is assumed that HC is of full rank and (HC,A) is an observable pair. Let's define

$$\psi(i)=H(C\tilde{x}(i)-y(i))\in R^p \tag{13}$$

equivalently

$$\psi(i)=HC\xi(i) \tag{14}$$

Equations (12) and (14) are considered the error system with the output $\psi(i)$ representing the dynamic behavior of the estimation error.

We now add the following dynamic system in the above observer.

$$w(i+1)=Pw(i)-F\psi(i) \tag{15.a}$$

$$\phi_j(i)=w_{j+1}(i)\ ; j=0,1,\cdots,k \tag{15.b}$$

where

$$w(i)=[w_1^T(i)\ w_2^T(i)\cdots w_L^T(i)]^T\in R^{pL} \tag{15.c}$$

and

$$F=[F_1^T\ F_2^T\ \cdots F_L^T]^T\in R^{pL\times p} \tag{15.d}$$

P in (15.a) has already been defined by (4.c). Consequently, the digital observer proposed here is shown by (10), (13) and (15). From (12), (14) and (15)

$$\begin{bmatrix}\xi(i+1)\\ w(i+1)\end{bmatrix}=\begin{bmatrix}A-EC & \Lambda_0\ \Lambda_1\cdots\Lambda_k\ 0\\ -FHC & P\end{bmatrix}\begin{bmatrix}\xi(i)\\ w(i)\end{bmatrix}-\begin{bmatrix}\Lambda_0\ \Lambda_1\ \cdots\Lambda_k\ 0\\ 0\end{bmatrix}q(i) \tag{16}$$

By defining

$$\zeta(i) = w(i) - q(i) \tag{17}$$

we obtain

$$\begin{bmatrix} \xi(i+1) \\ \zeta(i+1) \end{bmatrix} = \begin{bmatrix} A-EC & \Lambda_0\ \Lambda_1 \cdots \Lambda_k\ 0 \\ -FHC & P \end{bmatrix} \begin{bmatrix} \xi(i) \\ \zeta(i) \end{bmatrix} \tag{18}$$

Therefore, if (18) is stable, $\xi(i) \to 0$ can be attained at $i \to \infty$, i.e. $\tilde{\chi}(i) \to \chi(i)$ at $i \to \infty$. This means that the system-state is estimated with rejecting the periodic disturbance by the proposed observer. However, since the periodic disturbance (3.d) has been approximated by (7), the estimation accuracy depends on the degree of this approximation. The sampling period T and the order k of the extrapolation function arethe factors that we can adjust to improve the degree of the approximation.

Let's investigate the stability of (18). We represent E as

$$E = E_0 + \bar{E}H \tag{19}$$

where $E_0 \in R^{n \times m}$ and $\bar{E} \in R^{n \times p}$. We can write the coefficient matrix of (18) as follows:

$$\begin{bmatrix} \bar{A} & \Lambda_0\ \Lambda_1 \cdots \Lambda_k\ 0 \\ 0 & P \end{bmatrix} - \begin{bmatrix} \bar{E} \\ F \end{bmatrix} \begin{bmatrix} HC & 0 \end{bmatrix} \tag{20}$$

where $\bar{A} = A - E_0 C$. Since the system (3) is observerble, we can choose E_0 such that $\{\lambda_i\} \cap \{\lambda'_i\} = \phi$ and $\lambda'_i; i=0,1,\cdots,n$ are distinct, where $\{\lambda_i\}$ is the set of the zeros of $1 - z^L = 0$ and $\{\lambda'_i\}$ the set of the eigenvalues of $\bar{A}$. Therefore, it is found that if and only if

$$\left(\begin{bmatrix} HC & 0 \end{bmatrix}, \begin{bmatrix} \bar{A} & \Lambda_0\ \Lambda_1 \cdots \Lambda_k\ 0 \\ 0 & P \end{bmatrix} \right) \tag{21}$$

is an observable pair, the eigenvalues of (20) can be arbitrarily asigned by suitable design of $[\bar{E}^T\ F^T]^T$. Consequently, there is H satisfying (21) if and only if

$$\text{rank} \begin{bmatrix} A - zI & \sum_{j=0}^{k} \Lambda_j z^j \\ C & 0 \end{bmatrix} = n + p \ \ \text{for all } z \in \{\lambda_i\} \tag{22}$$

or equivalently

$$\text{the zeros of a system } \left\{ [C\ \ 0], \begin{bmatrix} A & \Lambda_0 & \Lambda_1 & \cdots & \Lambda_{k-1} \\ 0 & 0 & I_p & & 0 \\ & & 0 & \ddots & \\ \vdots & \vdots & & \ddots & I_p \\ 0 & 0 & \cdots & & 0 \end{bmatrix}, \begin{bmatrix} \Lambda_k \\ 0 \\ \vdots \\ 0 \\ I_p \end{bmatrix} \right\} \cap \{\lambda_i\} = \emptyset \tag{23}$$

where the notation $\{C, A, B\}$ denotes the system with the output matrix C, the system coefficient matrix A and the input matrix B. [See Appendix A] If the above condition is valid for a given system, we can easily find H satisfying (21) (or equivalently (A 3) and (A 4) in Appendix A) because almost all $p \times m$ constant matrices are such ones.

If the disturbance $q[t]$ is approximated by a step-wise function that is constant between sampling instants,i.e. $d(i)=\Lambda_0 q(i)$,the condition (22) will be shown as follows:

$$\mathrm{rank}\begin{bmatrix} A-zI_n & \Lambda_0 \\ C & 0 \end{bmatrix}=n+p \quad \text{for all } z\in\{\lambda_i\} \tag{24}$$

This result means that the zeros of the transfer function matrix between the disturbance $q(i)$ and the output $y(i)$ does not contain any $z\in\{\lambda_i\}$.

4. Disturbance rejection in a servomechanism

In this section, we investigate a design method of a servomechanism whose output tracks a reference signal with no steady-state error, together with the effective rejection of the periodic disturbance. Let $r(i)\in R^m$ be the reference signal that is not necessarily periodic. We introduce the following compensator to realize the output tracking to the reference signal with no steady-state error in case of no periodic disturbance.

$$v(i+1)=A_s v(i)+B_s(r(i)-y(i)) \tag{25}$$

where $v(i)\in R^n$,$A_s\in R^{n\times n}$ and $B_s\in R^{n\times m}$. This compensator includes the internal model of the reference signal generator. By choosing the input $u(i)$ as

$$u(i)=Gv(i)+Kx(i) \tag{26}$$

where $G\in R^{m\times n}$ and $K\in R^{m\times n}$, the total system is given by

$$x(i+1)=(A+BK)x(i)+BGv(i) \tag{27.a}$$

$$v(i+1)=-B_sCx(i)+A_sv(i)+B_sr(i) \tag{27.b}$$

$$y(i)=Cx(i) \tag{27.c}$$

under the assumptions of $d(i)=0$ and direct measurement of the state. It is assumed that G and K are determined so that the total system is stable. The control law stated above is well known as a basic design method to obtain the output tracking to $r(i)$ with no steady-state error. If there is a periodic disturbance $d(i)$, $y(i)$ is inevitably affected by $d(i)$ since $d(i)$ generally spans an n-dimensional vector space as explained in §2. Although $\tilde{x}(i)$ estimated by the observer derived in the previous section is used, this effect cannot be removed from $y(i)$ because $x(i)$ itself is affected by $d(i)$. A method to reject the effect is to add not only the internal model of the reference signal generator but also that of the periodic disturbance generator in(25). However, in this case the transient error caused by a nonperiodic reference signalcan periodically appear for a long time. To avoid this situation, a new compensation scheme is investigated by applying the result obtained in the previous section.

We now introduce the following observer-type compensator:

$$\tilde{x}(i+1)=A\tilde{x}(i)-E(C\tilde{x}(i)-y(i))+Bu(i)+B\phi_0(i) \tag{28}$$

where $\tilde{x}(i)\in R^n$ and $E\in R^{n\times m}$. We choose $u(i)$ as

$$u(i) = Gv(i) + K\tilde{x}(i) - \phi_0(i) \tag{29}$$

$\phi_0(i) \in R^m$ in (28) and (29), which is discussed later, is an external input introduced to reject the periodic disturbance. Let

$$\xi(i) = \tilde{x}(i) - x(i) \in R^n \tag{30.a}$$

and

$$\psi(i) = C\tilde{x}(i) - y(i) \in R^m \tag{30.b}$$

Therefore, the following relation holds.

$$\xi(i+1) = (A-EC)\xi(i) - d(i) + B\phi_0(i) \tag{31.a}$$

$$\psi(i) = C\xi(i) \tag{31.b}$$

On the other hand, from (3),(25),(28),(29) and (30)

$$\tilde{x}(i+1) = (A+BK)\tilde{x}(i) + BGv(i) - EC\xi(i) \tag{32.a}$$

$$v(i+1) = -B_sC\tilde{x}(i) + A_sv(i) + B_sr(i) + B_sC\xi(i) \tag{32.b}$$

$$y(i) = C\tilde{x}(i) - C\xi(i) \tag{32.c}$$

Consequently, the total system is described by (31) and (32). We can see that if $C\xi(i) \to 0$ $(i \to \infty)$ is realized in (32), the effect of the periodic disturbance can gradually be zeroed in (32). If (32) is stable, (32) finally behaves as the ideal servomechanism (27) whose output $y(i)$ tracks the reference signal with no steady-state error although the controlled system is disturbed by the periodic disturbance. The stability of (32) is always guaranteed by suitable design of G and K.

The remaining problem is to derive a compensation scheme attaining $C\xi(i) \to 0$ $(i \to \infty)$. Since $d(i)$ generally varies in an n-dimensional vector space, it is evident that $\xi(i) \to 0$ $(i \to \infty)$ cannot be realized whatever we choose in (31). We use the same dynamic system as (15), where we choose $k=0$ and alter integer p into m. We add this system in (28). Therefore,the observer-type compensator is augmented by this addition. From (15) and (31).

$$\begin{bmatrix} \xi(i+1) \\ w(i+1) \end{bmatrix} = \begin{bmatrix} A-EC & B & 0 \\ -FC & & P \end{bmatrix} \begin{bmatrix} \xi(i) \\ w(i) \end{bmatrix} + \begin{bmatrix} -I_m \\ 0 \end{bmatrix} d(i) \tag{33.a}$$

$$\psi(i) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \xi(i) \\ w(i) \end{bmatrix} \tag{33.b}$$

It is easily derived that (33) can be stabilized by suitable design of E and F if and only if

$$\text{rank} \begin{bmatrix} A - zI_n & B \\ C & 0 \end{bmatrix} = n + m \text{ for all } z \in \{\lambda_i\} \tag{34}$$

It is assumed that the system satisfies this condition. We now use (8) as the generator of the periodic disturbance $d(i)$ in this section. Then, from (8) and (33)

$$\begin{bmatrix} \xi(i+1) \\ w(i+1) \\ \eta(i+1) \end{bmatrix} = \begin{bmatrix} A-EC & B & 0 & -D \\ -FC & & P & 0 \\ 0 & & 0 & \bar{P} \end{bmatrix} \begin{bmatrix} \xi(i) \\ w(i) \\ \eta(i) \end{bmatrix} \tag{35.a}$$

$$\psi(i) = \begin{bmatrix} C & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi(i) \\ w(i) \\ \eta(i) \end{bmatrix} \tag{35.b}$$

If (33) is stable, the unstable modes of (35.a) come from the eigenvalues of $\bar{P}$. However, wecan show that the modes corresponding to these eigenvalues $\{\lambda_i\}$ are unobservable. [See Appendix B] Therefore, we can see that if (33) is stable, $\psi(i)=C\xi(i)\to 0$ $(i\to\infty)$ can be attained.

5. Signals generated by the observer-type compensator in steady state

The servomechanism derived in the previous section works with the desirable performance: output tracking and periodic-disturbance rejection. In this section, the steady-state value of $\phi_0(i)$ that directly relates to the elimination of the periodic disturbance is considered.

From (31)

$$\psi(z) = C(zI - A + EC)^{-1}(B\phi_0(z) - d(z)) \tag{36}$$

where $\psi(z), \phi_0(z)$ and $d(z)$ are the z-transform of the sequences $\{\psi(i)\}, \{\phi_0(i)\}$ and $\{d(i)\}$. Equation (36) is further described as

$$N(z)\psi(z) = M(z)\phi_0(z) - \hat{M}(z)d(z) \tag{37}$$

where $N(z) \in R^{m\times m}[z]$, $M(z) \in R^{m\times m}[z]$ and $\hat{M}(z) \in R^{m\times m}[z]$. We can choose $N(z)$ and $M(z)$ such that they are relatively left prime and $M(z)$ is row proper. $M(z)$ and $\hat{M}(z)$ have the following forms:

$$M(z) = M_0 z^{\nu} + M_1 z^{\nu-1} + \cdots + M_{\nu} \tag{38.a}$$

$$\hat{M}(z) = \hat{M}_0 z^{n-1} + \hat{M}_1 z^{n-2} + \cdots + \hat{M}_{n-1} \tag{38.b}$$

where $M_i \in R^{m\times m}$; $i=0,1,\cdots$, and $\hat{M}_i \in R^{m\times n}$; $i=0,1,\cdots,n-1$ and $\nu\ (\leq n-1)$ is the maximum integer such that $M_0 \neq 0$.

We define the signal sequences included in a repetitive period L as the following vectors:

$$\Phi_0(i) \triangleq \left[\phi_0{}^T(i-1) \quad \phi_0{}^T(i-2) \quad \cdots \quad \phi_0{}^T(i-L)\right]^T \in R^{mL} \tag{39.a}$$

$$\Delta(i) \triangleq \left[d^T(i-1) \quad d^T(i-2) \quad \cdots \quad d^T(i-L)\right]^T \in R^{nL} \tag{39.b}$$

where $i \geq L$. Once $\psi(i)=0$ is attained at $i\to\infty$ as mentioned in the previous section, the following relation holds.

$$\mathbf{M}\Phi_0(i) = \hat{\mathbf{M}}\Delta(i) \tag{40.a}$$

where $M \in R^{mL \times mL}$ and $\hat{M} \in R^{mL \times nL}$ are given by

$$\mathbf{M} = \begin{bmatrix} M_0 & M_1 & M_2 & \cdots & M_v & 0 & 0 & \cdots & 0 \\ 0 & M_0 & M_1 & M_2 & \cdots & M_v & 0 & \cdots & 0 \\ 0 & 0 & M_0 & M_1 & M_2 & \cdots & M_v & \cdots & 0 \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \\ \cdots & M_v & 0 & 0 & \cdots & 0 & M_0 & M_1 & M_2 \\ M_2 & \cdots & M_v & 0 & 0 & \cdots & 0 & M_0 & M_1 \\ M_1 & M_2 & \cdots & M_v & 0 & 0 & \cdots & 0 & M_0 \end{bmatrix} \tag{40.b}$$

and

$$\hat{\mathbf{M}} = \begin{bmatrix} \hat{M}_{n-v-1} & \hat{M}_{n-v} & \cdots & \hat{M}_{n-1} & 0 & \cdots & 0 & \hat{M}_0 & \cdots & \hat{M}_{n-v-2} \\ \hat{M}_{n-v-2} & \hat{M}_{n-v-1} & \hat{M}_{n-v} & \cdots & \hat{M}_{n-1} & 0 & \cdots & 0 & \hat{M}_0 & \cdots \\ \hat{M}_{n-v-3} & \hat{M}_{n-v-2} & \hat{M}_{n-v-1} & \hat{M}_{n-v} & \cdots & \hat{M}_{n-1} & 0 & \cdots & 0 & \cdots \\ & & & & \vdots & & & & & \\ & & & & \vdots & & & & & \\ \cdots & 0 & \cdots & 0 & \hat{M}_0 & \hat{M}_1 & \cdots & \hat{M}_{n-v-1} & \hat{M}_{n-v} & \hat{M}_{n-v+1} \\ \cdots & \hat{M}_{n-1} & 0 & \cdots & 0 & \hat{M}_0 & \hat{M}_1 & \cdots & \hat{M}_{n-v-1} & \hat{M}_{n-v} \\ \hat{M}_{n-v} & \cdots & \hat{M}_{n-1} & 0 & \cdots & 0 & \hat{M}_0 & \hat{M}_1 & \cdots & \hat{M}_{n-v-1} \end{bmatrix} \tag{40.c}$$

Therefore, we finaly obtain

$$\Phi_0(i) = \mathbf{M}^{-1}\hat{\mathbf{M}}\Delta(i) \tag{41}$$

if M is nonsingular. It is found that $\phi_0(i)$ is uniquely determined as a function of the periodic disturbance in steady state. If $D_c = B_c$ and $q[t]$ can be assumed constant between sampling instants, i.e. $d(i) = Bq[iT]$, $\phi_0(i) = q[iT]$ holds in steady state. In this case, $\phi_0(i)$ is the estimation value of the periodic disturbance. The necessary and sufficient condition for the nonsingularity of M is equivalent to the existence condition (34) of the observer-type compensator. [See Appendix C]

6. Simulation results

Consider the system shown in Fig.1, in which two carts are coupled by a spring and a damper. The carts can travel on the floor that periodically moves right and left. We assume that slip does not occur between the wheels and the floor. This system is modeled by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -K_0/M_1 & -(D_0+D_1)/M_1 & K_0/M_1 & -D_0/M_1\\ 0 & 0 & 0 & 1\\ K_0/M_2 & D_0/M_2 & -K_0/M_2 & -(D_0+D_2)/M_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\0\\0\\1\end{bmatrix}u+\begin{bmatrix}0\\ D_1/M_1\\ 0\\ D_2/M_2\end{bmatrix}\dot{h}$$

$y_1=x_1$ and $y_2=x_3$ (42)

where x_1 and x_2 are the position andthe velocity of the left cart, x_3 and x_4 those of the right cart, h the position of the floor and u the control input force. The values of the parameters are given as the mass $M_1=4$ and $M_2=2$(kg), the viscous friction

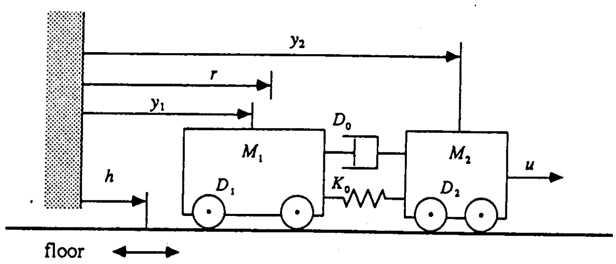


Fig.1 Two carts coupled by a spring and a damper on a moving floor

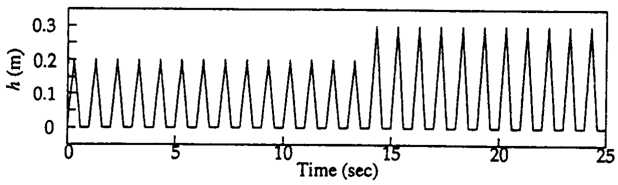


Fig.2 Movement of the floor

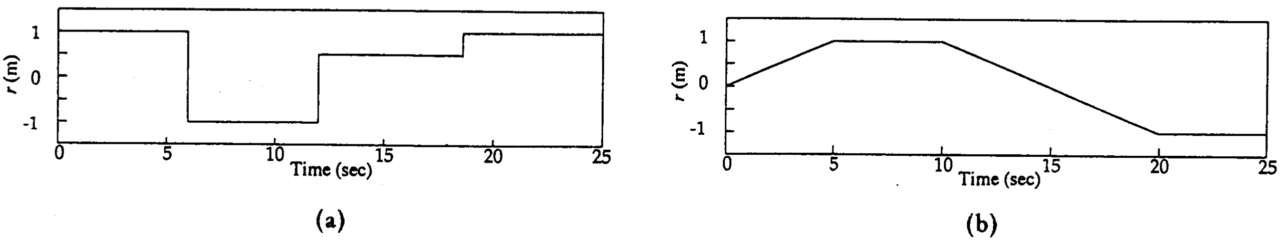
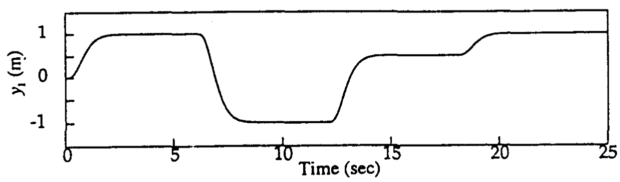
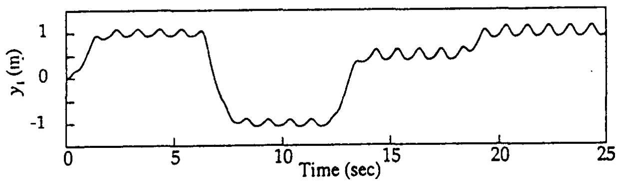


Fig.3 Two patterns of position reference signals

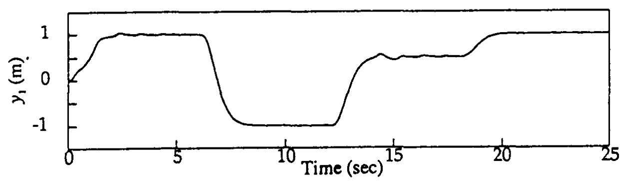
coefficients $D_0=10$, $D_1=20$ and $D_2=10$(Ns/m) and the spring constant $K_0=100$(N/m). The purpose is to control the force so that the left cart (position y_1) tracks the position reference-signal r as precisely as possible under the circumstances that the carts are shaked by the moving floor. The floor moves as shown in Fig.2. Two patterns of the position reference-signals given in Fig.3 have been adopted. We have chosen the sampling period 0.05(sec). To realize output tracking with no steady-state error to the position reference-signal of a step function, a compensator : $v(i+1)=v(i)+(r(i)-y(i))$ corresponding to (25) has been introduced. The feedback gain matrices G and K in (6) have been determined so that the poles of the servomechanism are assigned at 0.76, 0.77, 0.78, 0.79 and 0.80. We have chosen the gain matrices E in (8) and F in (13.b) such that the performance index $J=10^3\times\sum_{i=0}^{\infty}\psi^2(i)+\sum_{i=0}^{\infty}\phi_0^2(i)$ is minimized. Fig.4 and Fig.5 show that the influence of the moving floor is gradually rejected and the same output tracking property as that of the ideal servomechanism is finally realized.



(a) Conventional servomechanism
without the periodic disturbance

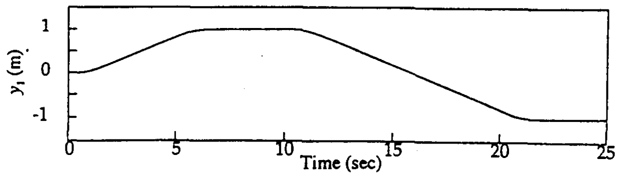


(b) Conventional servomechanism
with the periodic disturbance

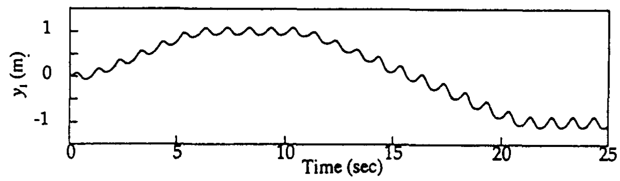


(c) Proposed sevomechanism
with the periodic disturbance

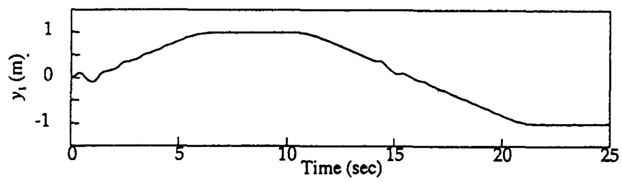
Fig.4 Output patterns to the reference in Fig.3 (a)



(a) Conventional servomechanism
without the periodic disturbance



(b) Conventional servomechanism
with the periodic disturbance



(c) Proposed sevomechanism
with the periodic disturbance

Fig.5 Output patterns to the reference in Fig.3 (b)

7. Conclusions

We have investigated digital observers and servomechanisms rejecting periodic disturbances. Their own characteristics required as observers and servomechanisms: state estimation and output tracking are not affected by the disturbance rejection. We can arbitrarily specify the rates of convergence of the estimation errors and tracking errors caused by the periodic disturbances. The order of the observer is the sum of orders of the system and the disturbance generator. The observer-type compensator introduced in the servomechanism has also the same order. If there is no periodic distubance, the dynamic behavior of the observer and the servomechanism is absolutely identical with that of conventional ones.

Appendix A

The observability of (21) can be confirmed by

$$\operatorname{rank}\begin{bmatrix} \bar{A}-zI_n & \Lambda_0 & \Lambda_1 & \cdots & \Lambda_k & 0 \\ 0 & & & P-zI_{Lp} & & \\ HC & & & 0 & & \end{bmatrix} = n+(L+1)p \tag{A 1}$$

for all $z\in\{\{\lambda_i\}\cup\{\lambda_i'\}\}$. By using the elementary column and row operations, condition (A 1) is simplified as follows:

$$\mathrm{rank}\begin{bmatrix}\bar{A}-zI_n & \sum_{j=0}^{k}\Lambda_j z^j \\ 0 & I_p-z^{-L}I_p \\ HC & 0\end{bmatrix}=n+p \qquad \text{(A 2)}$$

for all $z\in\{\{\lambda_i\}\cup\{\lambda_i'\}\}$. (A 2) can further be simplified as follows:

$$\mathrm{rank}\begin{bmatrix}\bar{A}-zI_n \\ HC\end{bmatrix}=n \text{ for all } z\in\{\lambda_i'\}\text{,i.e. the pair } (HC,\bar{A}) \text{ is observable} \qquad \text{(A 3)}$$

since $\mathrm{rank}(I_p-z^L I_p)=p$ for all $z\in\{\lambda_i'\}$ and

$$\mathrm{rank}\begin{bmatrix}\bar{A}-zI & \sum_{j=0}^{k}\Lambda_j z^j \\ HC & 0\end{bmatrix}=n+p \text{ for all } z\in\{\lambda_i\} \qquad \text{(A 4)}$$

since $I_p-z^L I_p=0$ for all $z\in\{\lambda_i\}$. Since the pair (C,A) is observable and $\mathrm{rank}[C^T,\bar{A}^T]=\mathrm{rank}[C^T,A^T]$, there is always H satisfying (A 3). Consequently, we can see that there is H satisfying (A 4) if and only if (22) is valid. It is evident that (23) is equivalent to (22).

Appendix B

Note that (34) is satisfied and $\lambda_i; i=1,2,\cdots,L$ are distinct since they are the zeros of $1-z^L=0$. If the mode corresponding to $z\in\{\lambda_i\}$ is observable,

$$\mathrm{rank}\begin{bmatrix}A-EC-zI_n & B \quad 0 & -D \\ -FC & P-zI_{mL} & 0 \\ 0 & 0 & \bar{P}-zI_L \\ C & 0 & 0\end{bmatrix}=n+(m+1)L \qquad \text{(A 5)}$$

for this z. Transforming the above matrix by the elementary row and column operations, the condition is equivalently shown as follows:

$$\mathrm{rank}\begin{bmatrix}A-zI_n & B & -\sum_{i=1}^{L}D_i z^{i-1} & 0 \\ C & 0 & 0 & 0 \\ 0 & I_m-z^L I_m & 0 & 0 \\ 0 & 0 & 1-z^L & 0 \\ 0 & 0 & 0 & I_{(m+1)(L-1)}\end{bmatrix}=n+(m+1)L \qquad \text{(A 6)}$$

where $D=[D_1 D_2\cdots D_L]$. Since $I_m-z^L I_m=0$ and $1-z^L=0$ for this z, (A 6) is not valid, i.e. the rank is $n+(m+1)L-1$. Therefore, we can see that this mode is not observable.

Appendix C

Let ν_i be the highest degree occurring among the elements in the i th row of $M(z)$. Therefore, $\nu = \max_i\{\nu_i\}$. We further define

$$\overline{M}(z) = \overline{M}_0 z^{\nu} + \overline{M}_1 z^{\nu-1} + \cdots + \overline{M}_{\nu} \tag{A 7}$$

, which is described by

$$\overline{M}(z) = \mathrm{diag}[z^{\nu-\nu_1}\ z^{\nu-\nu_2} \cdots z^{\nu-\nu_m}]M(z) \tag{A 8}$$

where $\overline{M}_i \in R^{m\times m}$; $i=0,1,\cdots,\nu$. Since $\overline{M}_0$ is nonsingular, we assume that $\overline{M}_0 = I_m$. This can always be satisfied by premultiplying both sides of (37) by a suitable unimodular matrix.

The coprimeness of det$M(z)$ and z^L-1 is equivalent to the condition (34). As seen from the definition of $\overline{M}(z)$, the coprimeness of det$M(z)$ and z^L-1 is also equivalent to that of det$\overline{M}(z)$ and z^L-1.The necessary and sufficient condition of the coprimeness of det$\overline{M}(z)$ and z^L-1 is that

$$\begin{matrix} mL \left\{ \begin{matrix} \\ \\ \\ \\ \end{matrix} \right. \\ m\nu \left\{ \begin{matrix} \\ \\ \\ \end{matrix} \right. \end{matrix}
\begin{bmatrix}
\overline{M}_0 & \overline{M}_1 & \overline{M}_2 & \cdots & \overline{M}_\nu & 0 & & \cdots & \cdots & 0 \\
0 & \overline{M}_0 & \overline{M}_1 & \overline{M}_2 & \cdots & \overline{M}_\nu & 0 & & \cdots \cdots & 0 \\
 & & & & & \vdots & & & & \\
 & & & & & \vdots & & & & \\
0 & & \cdots & \cdots & & 0 & \overline{M}_0 & \overline{M}_1 & \overline{M}_2 \cdots & \overline{M}_\nu \\
I_m & 0 & & \cdots & \cdots & & 0 & -I_m & 0 \cdots & 0 \\
0 & I_m & 0 & & \cdots & \cdots & & 0 & -I_m\ \ 0 & \cdots \\
 & & & & & \vdots & & & & \\
0 & \cdots & 0 & I_m & 0 & & \cdots & \cdots & 0 & -I_m
\end{bmatrix} \in R^{m(L+\nu)\times m(L+\nu)} \tag{A 9}$$

is nonsingular[15]. (A 9) is transformed into the following form:

$$\begin{bmatrix}
 & & \overline{\mathbf{M}} & & & & & 0_{mL\times m\nu} & & \\
I_m & 0 & & \cdots & \cdots & 0 & -I_m & 0 & \cdots & 0 \\
0 & I_m & 0 & & \cdots \cdots & & 0 & -I_m & 0 & \cdots \\
 & & & & \vdots & & & & & \\
0 & \cdots & 0 & I_m & 0 & & \cdots \cdots & & 0 & -I_m
\end{bmatrix} \tag{A 10.a}$$

by the elementary row operations, where

$$\overline{\mathbf{M}} = \begin{bmatrix} \overline{M}_0 & \overline{M}_1 & \overline{M}_2 & \cdots & \overline{M}_v & 0 & 0 & \cdots & 0 \\ 0 & \overline{M}_0 & \overline{M}_1 & \overline{M}_2 & \cdots & \overline{M}_v & 0 & \cdots & 0 \\ 0 & 0 & \overline{M}_0 & \overline{M}_1 & \overline{M}_2 & \cdots & \overline{M}_v & \cdots & 0 \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \\ \cdots & \overline{M}_v & 0 & 0 & \cdots & 0 & \overline{M}_0 & \overline{M}_1 & \overline{M}_2 \\ \overline{M}_2 & \cdots & \overline{M}_v & 0 & 0 & \cdots & 0 & \overline{M}_0 & \overline{M}_1 \\ \overline{M}_1 & \overline{M}_2 & \cdots & \overline{M}_v & 0 & 0 & \cdots & 0 & \overline{M}_0 \end{bmatrix} \in R^{mL \times mL} \quad \text{(A 10.b)}$$

Therefore, if and only if $\overline{M}$ is nonsingular, so is (A 10.a). We can derive M defined by (40.b)by interchanging the rows of $\overline{M}$. Consequently, it is found that the nonsingularity of M is equivalent to the condition (34).

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(Received December 11,1997; Accepted February 3,1998)